أجابة نماذج نصار أمتحان تقييمى أول

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أولا الأسئلة المقالية

(1)

$$\int (x+2)\sqrt[3]{x^2+4x-1}\,dx$$

الحل :

بفرض $u = x^2 + 4x - 1$

$du = (2x+4)dx \quad , \quad \frac{1}{2}du = (x+2)dx$

$$\int (x+2)\sqrt[3]{x^2+4x-1}\,dx = \int u^{\frac{1}{3}}\left(\frac{1}{2}\,du\right)$$

$$= \frac{1}{2}\int u^{\frac{1}{3}}\,du$$

$$= \frac{1}{2}\left[\frac{3}{4}\,u^{\frac{4}{3}}\right] + C$$

$$\therefore \int (x+2)\sqrt[3]{x^2+4x-1}\,dx = \frac{3}{8}(x^2+4x-1)^{\frac{4}{3}} + C$$

(2)

أوجد :

$$\int \frac{5}{\sqrt{x}\,(\sqrt{x}+2)^3}\,dx$$

$$\int \frac{5}{x^{\frac{1}{2}}(x^{\frac{1}{2}}+2)^3}\,dx =$$

$$= \int 5\,x^{\frac{-1}{2}}\,(x^{\frac{1}{2}}+2)^{-3}\,dx = 5\int (x^{\frac{1}{2}}+2)^{-3}\,x^{\frac{-1}{2}}\,dx$$

نفرض أن $u = x^{\frac{1}{2}}+2$ تفاضل $du = \frac{1}{2}x^{\frac{-1}{2}}\,dx$

$$2\,du = x^{\frac{-1}{2}}\,dx$$

$$5\int (x^{\frac{1}{2}}+2)^{-3}\,x^{\frac{-1}{2}}\,dx = 5\int u^{-3}.2\,du = 10\int u^{-3}\,du$$

$$= 10.\frac{u^{-2}}{-2} + C = \frac{-5}{u^2} + C = \frac{-5}{(\sqrt{x}+2)^2} + C$$

(3)

$$\int x(2x-1)^3\,dx$$

الحل:

$$u=2x-1 \Rightarrow du=2dx \Rightarrow \frac{du}{2}=dx$$

$$u=2x-1 \Rightarrow 2x=u+1 \Rightarrow x=\frac{u+1}{2}$$

$$\int x(2x-1)^3\,dx=\int\left(\frac{u+1}{2}\right)u^3\frac{du}{2}=\frac{1}{4}\int(u^4+u^3)du$$

$$=\frac{1}{4}\left(\frac{u^5}{5}+\frac{u^4}{4}+C\right)=\frac{1}{20}(2x-1)^5+\frac{1}{16}(2x-1)^4+C$$

(4)

$$\int x^5\sqrt{3+x^2}\,dx$$

الحل:

$$u = 3+x^2 \Rightarrow du = 2xdx \Rightarrow \frac{du}{2} = xdx$$

$$\int x^5\sqrt{3+x^2}\,dx = \int \sqrt{3+x^2}\,(x^4)(xdx)$$

$$u = 3+x^2 \Rightarrow x^2 = u-3 \Rightarrow x^4 = (u-3)^2$$

$$\int x^5\sqrt{3+x^2}\,dx = \int \sqrt{3+x^2}\,(x^4)(xdx)$$

$$= \int \sqrt{u}\,(u-3)^2\frac{du}{2} = \frac{1}{2}\int u^{\frac{1}{2}}(u^2-6u+9)du = \frac{1}{2}\int\left(u^{\frac{5}{2}}-6u^{\frac{3}{2}}+9u^{\frac{1}{2}}\right)du$$

$$= \frac{1}{2}\left(\frac{u^{\frac{7}{2}}}{\frac{7}{2}}-\frac{6u^{\frac{5}{2}}}{\frac{5}{2}}+\frac{9u^{\frac{3}{2}}}{\frac{3}{2}}+C\right) = \frac{1}{7}u^{\frac{7}{2}}-\frac{6}{5}u^{\frac{5}{2}}+3u^{\frac{3}{2}}+C$$

$$= \frac{1}{7}(3+x^2)^{\frac{7}{2}}-\frac{6}{5}(3+x^2)^{\frac{5}{2}}+3(3+x^2)^{\frac{3}{2}}+C$$

(5)

$$\int x\sec^2(x^2+2)\,dx$$

الحل:

$$u=x^2+2 \Longrightarrow du=2xdx \Longrightarrow \frac{1}{2}du=xdx$$

$$\int xsec^2(x^2+2)dx=\int sec^2u\left(\frac{1}{2}du\right)$$

$$=\frac{1}{2}\int sec^2u\,du=\frac{1}{2}tan\,u+C=\frac{1}{2}tan\,(x^2+2)+C$$

(6)

$$\int \csc^5 x\ \cot x\,dx$$

الحل:

$$u=\csc x \Rightarrow du=-\csc x\cot x\,dx \Rightarrow -du=\csc x\cot x\,dx$$

$$\int \csc^5 x.\cot x\,dx=\int \csc^4 x.\csc x.\cot x\,dx=\int u^4(-du)$$

$$=-\int u^4\,du=-\frac{u^5}{5}+C=-\frac{1}{5}\csc^5 x+C$$

(7)

$$\int \cot x \, dx$$

الحل:

$$\int \cot x \, dx = \int \frac{\cos x}{\sin x} dx$$

$$u = \sin x \Rightarrow du = \cos x \, dx$$

$$\int \cot x \, dx = \int \frac{du}{u} = \ln|u| + C = \ln|\sin x| + C$$

(8)

$$\int \cos^3(2x-3) \cdot \sin(2x-3) \, dx$$

$$u = \cos(2x-3)$$

$$du = -2\sin(2x-3) \, dx$$

$$\frac{du}{-2} = \sin(2x-3) \, dx$$

$$= \int u^3 \frac{du}{-2}$$

$$= \frac{-1}{2} \cdot \frac{u^4}{4} + C = \frac{-1}{8}(\cos(2x-3))^4 + C$$

(9)

$$\int \frac{dx}{(\sin^2 x)\sqrt{1+\cot x}}\, dx$$

$$= \int \frac{1}{\sqrt{1+\cot x}} \cdot \frac{1}{\sin^2 x}\, dx$$

$$= \int (1+\cot x)^{-1/2} \cdot \csc^2 x\, dx$$

نفرض أن

$$u = 1+\cot x$$

$$du = -\csc^2 x\, dx$$

$$\therefore -\int (1+\cot x)^{-1/2} \cdot -\csc^2 x\, dx$$

بالتعويض

$$= -\int u^{-1/2}\, du$$

$$= -\frac{u^{1/2}}{\frac{1}{2}} + C$$

$$= -2\sqrt{u} + C$$

$$= -2\sqrt{1+\cot x} + C$$

(10)

$$\int \frac{3t^2 - 6t}{t^3 - 3t^2 + 8} dt$$

نفرض أن $u = t^3 - 3t^2 + 8 \longrightarrow du = (3t^2 - 6t)\ dt$

$$\int \frac{3t^2 - 6t}{t^3 - 3t^2 + 8} dx = \int \frac{1}{u} du = ln\,|u| + C$$

$$= ln\left|t^3 - 3t^2 + 8\right| + C$$

....

$$\int \frac{x^3 + 4}{x} dx$$

$$= \int (\frac{x^3}{x} + \frac{4}{x}) dx$$

$$= \int (x^2 + \frac{4}{x})\ dx$$

$$= \frac{x^3}{3} + 4\ ln\ |x| + C$$

(11)

$$\int \frac{x+1}{\sqrt[3]{x}+1}dx = \int \frac{\cancel{\left(\sqrt[3]{x}+1\right)}\left(\sqrt[3]{x^2}-\sqrt[3]{x}+1\right)dx}{\cancel{\left(\sqrt[3]{x}+1\right)}}$$

$a^3+b^3=(a+b)(a^2-ab+b^2)$

$$= \int \left(\sqrt[3]{x^2}-\sqrt[3]{x}+1\right)dx$$

$$= \int \left(x^{\frac{2}{3}}-x^{\frac{1}{3}}+1\right)dx$$

قاعدة الجمع والطرح

$$= \frac{3}{5}x^{\frac{5}{3}}-\frac{3}{4}x^{\frac{4}{3}}+x+C$$

$$= \frac{3}{5}x\sqrt[3]{x^2}-\frac{3}{4}x\sqrt[3]{x}+x+C$$

(12)

أوجد: $\int (x^2-2)e^{x^3-6x}\,dx$

الحل

$$I = \int e^{x^3-6x}\cdot(x^2-2)\,dx$$

$$= \int e^{u}\cdot \frac{1}{3}\,du$$

$$= \frac{1}{3}e^{u}+c$$

$$= \frac{1}{3}e^{x^3-6x}+c$$

$u = x^3-6x$

$du = (3x^2-6)dx$

$du = 3(x^2-2)dx$

$\frac{1}{3}du = (x^2-2)dx$

(13)

أوجد: $\int \frac{1}{x^2} e^{\frac{1}{x}} dx$

الحل

$I = \int e^{\frac{1}{x}} \cdot \frac{1}{x^2} dx$

$= -\int e^{u} \cdot du$

$= -e^{u} + c = -e^{\frac{1}{x}} + c$

$u = \frac{1}{x}$

$du = -\frac{1}{x^2} dx$

$-du = \frac{1}{x^2} dx$

(14)

أوجد: $\int \frac{e^x}{e^x+1} dx$

الحل

$I = \int \frac{1}{u} du$

$= \ln|u| + c$

$= \ln|e^x + 1| + c$

$u = e^x + 1$

$du = e^x dx$

(15)

$$\int (2\tan x - \csc^2 x)\, dx$$

$$\int tanx\, dx = \int \frac{sin\, x}{cos\, x} dx \qquad \begin{vmatrix} u = \cos x \\ du = -\sin x\, dx \end{vmatrix}$$

$$= -\int \frac{du}{u}$$

$$= -\, ln\, |u| + \mathrm{C}$$

$$= -\, ln\, |\ cos\, x\,| + \mathrm{C}$$

$$\therefore \mathrm{I} = -2\ln|\cos x| + \cot x + \mathrm{C}$$

(16)

$$\int \frac{x^4 - 27x}{x^2 - 3x}\,dx$$

$$\int \frac{x(x^3 - 27)}{x(x-3)}\,dx = \int \frac{\cancel{(x-3)}(x^2+3x+9)}{\cancel{(x-3)}}\,dx$$

$$= \int (x^2 + 3x + 9)\,dx$$

$$\frac{x^3}{3} + \frac{3x^2}{2} + 9x + c$$

(17)

$$\int \frac{x-1}{\sqrt{x}+1}\,dx$$

$$= \int \frac{(x-1)}{\sqrt{x}+1} \times \frac{\sqrt{x}-1}{\sqrt{x}-1}\,dx$$

$$= \int \frac{(x-1)(\sqrt{x}-1)}{(x-1)}\,dx$$

$$= \int (\sqrt{x} - 1)\,dx$$

$$= \frac{2}{3}x^{\frac{3}{2}} - x + c$$

ثانيا الأسئله الموضوعية

في التمارين (1—5)، ظلّل الدائرة (a) إذا كانت العبارة صحيحة و (b) إذا كانت العبارة خاطئة.

(1) $F(x) = x^{-3}$ هي مشتقة عكسية للدالة: $f(x) = -3x^{-4}$ — **(a)** (b)

(2) $\int (-x^{-3} + x - 1)dx = \frac{1}{2}x^{-2} + \frac{1}{2}x^2 - x + C$ — **(a)** (b)

(3) $\int \frac{1}{x^2}dx = \frac{1}{x} + C$ — (a) **(b)**

(4) إذا كانت: $f'(x) = \frac{1}{x^2} + x \;,\; f(2) = 1$ ، فإن: $f(x) = -\frac{1}{x} + \frac{1}{2}x^2 + \frac{1}{2}$ — (a) **(b)**

(5) إذا كانت: $F(0) = 400 \;,\; F(x) = \int (3x^2 - 12x + 15)dx$ ، فإن: $F(x) = x^3 + 6x^2 + 15x + 400$

(a) **(b)**

في التمارين (6—12)، ظلّل رمز الدائرة الدّال على الإجابة الصحيحة.

(6) $\int \frac{4}{3}\sqrt[3]{t^2}\,dt =$

(a) $\frac{3t^{\frac{5}{3}}}{5} + C$ **(b)** $\frac{4t^{\frac{5}{3}}}{5} + C$

(c) $\frac{4}{3}\sqrt[3]{t^5} + C$ (d) $4\sqrt[3]{t^5} + C$

(7) $\int \left(\sqrt[3]{x^2} + \frac{1}{\sqrt[3]{x^2}}\right)dx =$

(a) $\frac{3}{5}\sqrt[3]{x}(x^{\frac{4}{3}} + 5) + C$ (b) $\frac{3}{5}x^{\frac{2}{3}}(x^{-\frac{2}{3}} + 5) + C$

(c) $\frac{5}{3}\sqrt[3]{x}(x^{\frac{4}{3}} + 5) + C$ (d) $\frac{5}{3}x^{\frac{4}{3}}(x^{\frac{2}{3}} + 5) + C$

(8) إذا كان: $x = -1 \;,\; y = -5 \;,\; \frac{dy}{dx} = x^{-\frac{2}{3}}$ فإنّ y تساوي:

(a) $-\frac{x^2}{3} - \frac{14}{3}$ (b) $3x^{\frac{1}{3}} + 2$

(c) $3x^{\frac{1}{3}} - 2$ (d) $3x^{\frac{1}{3}}$

(9) $\int \frac{2x + 3}{\sqrt{x}}dx =$

(a) $\frac{3}{4}x^{\frac{3}{2}} + \frac{3}{2}x^{\frac{1}{2}} + C$ (b) $\frac{1}{3}x^{\frac{3}{2}} + 6x^{\frac{1}{2}} + C$

(c) $\frac{4}{3}x^{\frac{3}{2}} + 6x^{\frac{1}{2}} + C$ (d) $\frac{4}{3}x^{\frac{3}{2}} + \frac{1}{6}x^{\frac{1}{2}} + C$

(10) $\int \sqrt{x}(2+x^2)dx =$

a $\frac{4}{3}x^{\frac{3}{2}}+\frac{2}{7}x^{\frac{7}{2}}+C$

b $\frac{3}{4}x^{\frac{3}{2}}+\frac{7}{2}x^{\frac{7}{2}}+C$

c $\frac{1}{3}x^{\frac{3}{2}}+\frac{7}{2}x^{\frac{7}{2}}+C$

d $\frac{4}{3}x^{\frac{3}{2}}+\frac{7}{2}x^{\frac{7}{2}}+C$

(11) $\int \frac{2+\sqrt[3]{x^2}}{\sqrt{x}}dx =$

a $x^{\frac{1}{2}}+\frac{6}{7}x^{\frac{7}{6}}+C$

b $4x^{\frac{1}{2}}+\frac{6}{7}x^{\frac{7}{6}}+C$

c $x^{\frac{1}{2}}+\frac{7}{6}x^{\frac{7}{6}}+C$

d $4x^{\frac{1}{2}}+\frac{7}{6}x^{\frac{7}{6}}+C$

(12) $\int \left(\frac{x^2-4x+4}{x-2}+2\right)^2 dx =$

a x^2+C

b $2x+C$

c $\frac{x^2}{2}+2x+C$

d $\frac{1}{3}x^3+C$

في التمارين (5—1)، ظلّل الدائرة (a) إذا كانت العبارة صحيحة و(b) إذا كانت العبارة خاطئة.

(1) $\int x(x^2-1)^{10}dx = \frac{1}{18}(x^2-1)^9+C$ — a b

(2) $\int (x+1)\sqrt[3]{x^2+2x+3}\,dx = \frac{3}{8}\sqrt[3]{(x^2+2x+3)^4}+C$ — a b

(3) $\int \frac{dx}{\sqrt{3x-2}} = 2\sqrt{3x-2}+C$ — a b

(4) $\int (2x^2-1)(2x^3-3x+4)^5dx = \frac{1}{18}(2x^3-3x+4)^6+C$ — a b

(5) $\int x\sqrt[3]{x+2}\,dx = \frac{3}{7}(x+2)^{\frac{7}{3}}-\frac{3}{2}(x+2)^{\frac{4}{3}}+C$ — a b

في التمارين (12—6)، ظلّل رمز الدائرة الدّال على الإجابة الصحيحة.

(6) $\int x(x^2+2)^7dx =$

a $\frac{1}{16}(x^2+2)^8+C$

b $\frac{1}{4}(x^2+2)^8+C$

c $\frac{1}{12}(x^2+2)^6+C$

d $\frac{1}{3}(x^2+2)^6+C$

(7) $\int \frac{x-1}{\sqrt{x-1}}dx =$

a) $\frac{1}{3}(x-1)^{\frac{2}{3}}+C$

b) $\frac{2}{3}(x-1)^{\frac{3}{2}}+C$

c) $\frac{2}{3}(x-1)^{\frac{2}{3}}+C$

d) $\frac{3}{2}(x-1)^{\frac{2}{3}}+C$

(8) $\int \frac{dx}{\sqrt[3]{3x+1}} =$

a) $\frac{2}{9}(3x+1)^{\frac{2}{3}}+C$

b) $\frac{2}{3}(3x+1)^{\frac{2}{3}}+C$

c) $2(3x+1)^{\frac{2}{3}}+C$

d) $\frac{1}{2}(3x+1)^{\frac{2}{3}}+C$

(9) $\int \frac{(2+\sqrt{x})^{12}}{\sqrt{x}}dx =$

a) $\frac{13}{2}(2+\sqrt{x})^{13}+C$

b) $\frac{2}{13}(2+\sqrt{x})^{13}+C$

c) $\frac{1}{26}(2+\sqrt{x})^{13}+C$

d) $\frac{1}{22}(2+\sqrt{x})^{11}+C$

(10) $\int \frac{(x+1)}{\sqrt[3]{x^2+2x+3}}dx =$

a) $\frac{3}{4}\sqrt[3]{(x^2+2x+3)^2}+C$

b) $\frac{3}{2}\sqrt[3]{(x^2+2x+3)^2}+C$

c) $3\sqrt[3]{(x^2+2x+3)^2}+C$

d) $\frac{3}{4}\sqrt[3]{x^2+2x+3}+C$

(11) $\int \frac{x}{\sqrt{x+1}}dx =$

a) $\frac{3}{2}\sqrt{(x+1)^3}-2\sqrt{x+1}+C$

b) $\frac{2}{3}\sqrt{(x+1)^3}-\frac{1}{2}\sqrt{x+1}+C$

c) $\frac{2}{3}\sqrt{(x+1)^3}-2\sqrt{x+1}+C$

d) $\frac{2}{3}\sqrt{(x+1)^3}+2\sqrt{x+1}+C$

(12) إذا كانت: $F(x)=\int (x+1)(2x^2+4x-1)dx$ ، $F(-2)=\frac{9}{8}$، فإن: $F(x)$ تساوي:

a) $\frac{1}{8}(2x^2+4x-1)^2+\frac{5}{4}$

b) $\frac{1}{8}(2x^2+4x-1)^2+1$

c) $\frac{1}{4}(2x^2+4x-1)^2+1$

d) $4(2x^2+4x-1)^2-1$

في التمارين (1—5)، ظلّل الدائرة (a) إذا كانت العبارة صحيحة و(b) إذا كانت العبارة خاطئة.

(1) $\int \sec^2 x\,dx = \tan x + C$ — (a) ● (b)

(2) $\int \csc^2 x\,dx = \cot x + C$ — (a) (b) ●

(3) $\left(F'(x) = \sec^2 x\ ,\ F\left(\frac{\pi}{4}\right) = -1\right) \Longrightarrow F(x) = \tan x + 2$ — (a) (b) ●

(4) $(F'(x) = \cos x + \sin x\ ,\ F(\pi) = 1) \Longrightarrow F(x) = \sin x - \cos x$ — (a) ● (b)

(5) $(F'(x) = \sec x\ \tan x\ ,\ F(0) = 4) \Longrightarrow F(x) = \sec x + 3$ — (a) ● (b)

في التمارين (6—12)، ظلّل رمز الدائرة الدّال على الإجابة الصحيحة.

(6) الصورة العامة للمشتقة العكسية للدالة f حيث $f(x) = 8 + \csc x \cot x$ هي:

(a) $F(x) = 8x + \csc x + C$
(b) $F(x) = 8x - \cot x + C$
(c) ● $F(x) = 8x - \csc x + C$
(d) $F(x) = 8x + \cot x + C$

(7) $\int \csc(5x)\cot(5x)\,dx =$

(a) $\frac{1}{5}\csc(5x) + C$
(b) $\csc(5x) + C$
(c) $\frac{1}{5}\cot(5x) + C$
(d) ● $-\frac{1}{5}\csc(5x) + C$

(8) $\int \sqrt[3]{\cot x}\csc^2 x\,dx =$

(a) $\frac{3}{4}\sqrt[3]{(\cot x)^4} + C$
(b) ● $-\frac{3}{4}\sqrt[3]{(\cot x)^4} + C$
(c) $-\frac{3}{4}\sqrt[4]{(\cot x)^3} + C$
(d) $3\sqrt[3]{(\cot x)^4} + C$

(9) إذا كانت $y_{\theta=0} = -3$ ، $\frac{dy}{d\theta} = \sin\theta$ فإنّ y تساوي:

(a) $-\cos\theta$
(b) $2 - \cos\theta$
(c) ● $-2 - \cos\theta$
(d) $4 - \cos\theta$

(10) $\int \sec^5 x \tan x\,dx =$

(a) $\frac{5}{3}\sec^5 x + C$
(b) $\frac{1}{5}\sec^6 x + C$
(c) ● $\frac{1}{5}\sec^5 x + C$
(d) $-\frac{5}{3}\sec^5 x + C$

(11) $\int \frac{\csc^2 x}{\sqrt[3]{2 + \cot x}}\,dx =$

(a) $\frac{3}{2}(2 + \cot x)^{\frac{2}{3}} + C$
(b) ● $-\frac{3}{2}(2 + \cot x)^{\frac{2}{3}} + C$
(c) $-2\sqrt{2 + \cot x} + C$
(d) $\frac{4}{3}(2 + \cot x)^{\frac{4}{3}} + C$

(12) $\int \frac{\sin(4x)}{\cos^5(4x)}\,dx =$

(a) $-\frac{1}{16}\cos^{-4}(4x) + C$
(b) ● $\frac{1}{16}\cos^{-4}(4x) + C$
(c) $-\cos^{-4}(4x) + C$
(d) $\cos^{-4}(4x) + C$

في التمارين (1—6)، ظلّل الدائرة (a) إذا كانت العبارة صحيحة و (b) إذا كانت العبارة خاطئة.

(1) إذا كانت: $y = 4^{x-2}$ فإن: $\frac{dy}{dx} = 4x$ — (a) (b)

(2) إذا كانت: $f(x) = e^{x^2}$ فإن: $f'(x) = 2xe^{2x}$ — (a) (b)

(3) إذا كانت: $g(x) = \ln(2x+2)$ فإن: $g'(x) = \frac{1}{2x+2}$ — (a) (b)

(4) إذا كانت: $y = x\ln x - x$ فإن: $y' = \ln x$ — (a) (b)

(5) $\int \frac{1}{2x} dx = \frac{\ln x}{2} + C$ — (a) (b)

(6) $\int \frac{1}{3x+1} dx = \ln(3x+1) + C$ — (a) (b)

في التمارين (7—14)، ظلّل رمز الدائرة الدّال على الإجابة الصحيحة.

(7) إذا كانت $y = e^{-5x}$، فإنّ $\frac{dy}{dx}$ تساوي:

(a) e^{-5x} (b) $-e^{-5x}$

(c) $-5e^{-5x}$ (d) $5e^{-5x}$

(8) إذا كانت $y = x^2e^x - xe^x$، فإنّ $\frac{dy}{dx}$ تساوي:

(a) $e^x(x^2+x-1)$ (b) $e^x(x^2-x)$

(c) $2xe^x - e^x$ (d) $e^x(x^2+2x+1)$

(9) إذا كانت $y = (\ln x)^2$، فإنّ $\frac{dy}{dx}$ تساوي:

(a) $\frac{\ln x}{x}$ (b) $\frac{2\ln x}{x}$

(c) $\frac{x\ln x}{2}$ (d) $\frac{2\ln^2 x}{x}$

(10) إذا كانت $y = \ln\left(\frac{10}{x}\right)$، فإنّ $\frac{dy}{dx}$ تساوي:

(a) $-\frac{10}{x}$

(b) $\frac{10}{x}$

(c) $\frac{1}{x}$

(d) $-\frac{1}{x}$

(11) إذا كانت $y = \ln(x^2+1)$، فإنّ $\frac{dy}{dx}$ تساوي:

(a) $\frac{x}{x^2+1}$

(b) $\frac{2}{x^2+1}$

(c) $\frac{2x}{x^2+1}$

(d) $-\frac{2x}{x^2+1}$

(12) $\int \frac{2x}{x^2+1}\,dx =$

(a) $2\ln(x^2+1)+C$

(b) $\ln(x^2+1)+C$

(c) $\frac{x^2}{x^2+1}+C$

(d) $\frac{x}{\frac{1}{3}x^2+1}+C$

(13) $\int \frac{e^x+e^{-x}}{2}\,dx =$

(a) $\frac{e^x-e^{-x}}{2}+C$

(b) $\frac{e^x+e^{-x}}{2}+C$

(c) $\frac{e^{-x}-e^x}{2}+C$

(d) $\frac{e^{2x}-e^{-2x}}{2}+C$

(14) $\int \frac{e^x}{e^x-4}\,dx =$

(a) $-\frac{1}{2}(e^x-4)+C$

(b) $\ln|e^x-4|+C$

(c) $-\ln|e^x-4|+C$

(d) $\frac{1}{2}\ln|e^x-4|+C$